LexingtonSO Science Olympiad
2026 Satellite Invitational

April 13th - 27th, 2026

Astronomy C

Answer Key

Directions:

- Each team will be given 50 minutes to complete this exam
- 3 sections: Section A (General), Section B (DSOs). Section C (Astrophysics)
- Use 2-3 decimal places in final answers for this test, partial credit will be given for work
- Tiebreakers (in order): Section C: 1a, 2d; Section A: 14, 25; Section B: 28
- Good luck!

Section A (General)

(34 points total)

1) Which of the following stages of stellar evolution is the Sun currently in? [0.5]
    a) Pre-Main Sequence
    b) Main Sequence
    c) Red Giant
    d) White Dwarf
2) Between which temperatures are HI regions most unstable? [1]
    a) 1 K - 10 K
    b) 10 K - 10,000 K
    c) 10,000 K - 100,000 K
    d) None of the above
3) [Select all that apply] At the turnoff point, a star: [1]
    a) Begins core hydrogen fusion
    b) Concludes core hydrogen fusion
    c) Begins shell hydrogen fusion
    d) Concludes shell hydrogen fusion
4) Briefly describe the Eddington Valve mechanism. What is the more modern term for this process? [2] Higher temps -> higher opacity[0.5] -> harder for radiation to escape [0.5] -> contraction -> higher temps (luminosity pulsates bc of this) [0.5]. Kappa mechanism [0.5]
5) Which of the following is a bright patch of nebulosity associated with protostars? [0.5]
    a) Herbig-Haro Object
    b) Herbig Ae/Be star
    c) Molecular Cloud
    d) Planetary Nebula
6) In which astronomical body is iron rain most prevalent? Why? [2] Brown Dwarf [+0.5], they can't sustain fusion [+0.5] that would destroy the iron [+1].
7) For each of the following variable stars, identify which stage of stellar evolution they are in. [2]
    a) Mira Asymptotic Giant Branch [+0.5]
    b) RR Lyrae Horizontal Branch [+0.5]
    c) PNNV White Dwarf [+0.5]
    d) T Tauri Pre-Main Sequence [+0.5]
8) Compared to typical stars of their type on an HR diagram, blue stragglers are located: [1]
    a) Up and to the left
    b) Up and to the right

c) Down and to the left

d) Down and to the right

9) Briefly describe one theory for the existence of blue straggler stars. [1.5]

Two stars that survive a binary star merger

10) In AGB stars, which nucleosynthesis process primarily fuses elements heavier than iron? [1]

S-process

11) Briefly describe the changes in a star's internal structure that lead to a dredge-up. [1.5] The star becomes fully convective [1], allowing deeper fusion products to be brought to the surface [0.5].

12) Neutron stars primarily slow their rotation by converting what? [1.5]

Rotational energy into spindown luminosity

13) If the opacity of a stellar layer in a radially pulsating star increases, how does this affect pulsation period and why? [1.5]

Pulsation period increases since the Kappa mechanism increases in strength

14) Generally the pressure inside a star is proportional to $P \propto M^{\alpha}/R^{\beta}$. Through dimensional analysis, determine α and β. [1.5]

$P \propto \frac{M^2}{R^4}$

15) Repeat question 14 by for the gravitational binding energy U for a star. [1.5]

$U \propto \frac{M^2}{R}$

16) What does the reduced mass approach for the two-body problem do? [1.5]

Keep the reference frame at the center of one body, reduces two-body problem to a one-body problem

17) On a parabolic orbit, what is the velocity along any point of the parabola equal to? [1.5]

Escape Velocity

18) In the cores of massive stars, what is the dominant form of pressure for hydrostatic equilibrium? [1.5]

Radiation Pressure

19) Around 1920, Sir Arthur Eddington showed that the period of pulsation P for a variable star is proportional to what? [1.5]

One over square root of mean density: $1/\sqrt{\rho}$

20) The main sequence turnoff point of a star cluster allows you to estimate what? [1.5]

Age of the star cluster

21) Knowing the B-V color index of a star, what property can be determined? [1.5]

Temperature

22) What theorem allows us to estimate the velocity dispersions of a star cluster? [1]

a) Jean's Mass Theorem
b) Stoke's Theorem
c) Virial Theorem
d) Schwarzkopf's Theorem

23) What is true about eclipsing binary stars? [1]

a) All eclipsing binary stars are mass transfer systems
b) Observations of them are biased towards close separation binary systems
c) All eclipsing binary stars are unstable and spiral inwards
d) Observations of these systems can reveal their age

24) Hypothetically, if black dwarfs existed in the current state of the universe, what wavelength would they be brightest in? [1]

a) Radio
b) Microwave
c) Infrared
d) Visible

25) For convective transport in a star to occur, the temperature gradient must be greater than what type of temperature gradient for the star? [2]

a) Adiabatic
b) Isentropic
c) Diathermic
d) Isothermal

Section B (DSOs)

(40 pts total)

Each of the following 7 questions will contain 4 subparts, all covering this year's DSOs.

26) This DSO served as the first proof that supernovae introduce phosphorus into the ISM. [6]

a) Identify this DSO. [0.5] Cassiopeia A

b) Identify the image(s) and wavelength(s) corresponding to this DSO. [2] Image 4 [0.5], Infrared [0.5] and Image 5 [0.5], Infrared [0.5]

c) Which specific supernova produced this DSO? Why is this type of supernova named the way it is? [1.5] Type IIb [0.5], because weak hydrogen line that eventually disappears [0.5] and second peak resembling Type Ib supernova [0.5]

d) Within this DSO's bubble-like shells, what process is responsible for generating white colors (in a near-IR image)? Explain how this process occurs. [2] Synchrotron Radiation [1], relativistic electrons [0.5] travel in curved paths due to magnetic field lines [0.5]

27) Hope you learned your history… [4]

a) Identify the DSO in image 6. [0.5] Tycho's SNR

b) Which other images are also images of this DSO? [1] Image 7 [0.5] and Image 10 [0.5]

c) Identify two scientists important in this DSO's history. [1] Varies, [0.5] per

d) If this DSO has a peak apparent magnitude of -4, calculate its distance, in parsecs. [1.5] 10^((-4+19.3+5)/5) = 11,480 pc (DO NOT ACCEPT 3.8 kpc, +0.5 for M=-19.3 for Type 1a, +0.5 for distance modulus, +0.5 for answer)

28) Electron Degeneracy Pressure is crucial in supporting stellar interiors. [4]

a) Identify three DSOs on this year's list that are supported by electron degeneracy pressure. [1] Janus, WDJ181058.67+311940.94, Mira (+0.33 per)

b) Which DSO from part (a) has the highest mass? In solar masses, what's its value? [1] WDJ181058.67+311940.94 [0.5], 1.555 solar masses [0.5]

c) Which of the DSOs from part (a) will one day be brighter than the Moon? [0.5] WDJ181058.67+311940.94

d) Briefly describe the connection between inverse beta decay and electron degeneracy pressure. [1.5] When electron degeneracy pressure is overcome [0.5], electrons are able to react with protons [0.5] to form neutrons and neutrinos (inverse beta decay) [0.5]

29) 650 light years away, this DSO represents the later stages of stellar evolution. [6]

a) Identify this DSO. [0.5] Helix Nebula

b) Identify the image(s) and wavelengths(s) corresponding to this DSO. [2] Image 15 [0.5], [+0.5] for not mentioning any other images. Optical [0.5] and Near-IR [0.5]

c) For each of the following lines present within this DSO, identify the corresponding molecule: 1) 6563 Å, 2) 6583 Å, 3) 5007 Å [1.5] 1) Hydrogen [0.25] alpha [0.25], 2) Nitrogen [0.25] II [0.25], 3) Oxygen [0.25] III [0.25]

d) Briefly describe two theories for the observed variation in this DSO's light curve. [2] Intrinsic stellar variability [1] or orbiting planet [1] or accretion of a former planet [1] (Max 2 points)

30) 550 light years away, this DSO exhibits both periodic and random fluctuations. [6]

a) Identify this DSO. [0.5] HP Tau

b) Approximately how old is this DSO? Why might this be important to astronomers? [1] 5 million years [+0.5], insight into low mass young star formation [+0.5]

c) Identify two ways in which astronomers could conclude this is not a binary system. [1] Additional mass indicates 3rd companion [0.5], three stars visible [0.5]

d) At 3.4 and 1.2 mm respectively, identify the spectral index of this DSO. What do the scale of these values indicate about this DSO? [3.5] 9.4 [1] and 62 [1], can't make a conclusion [1] because not observed at radio wavelengths [0.5]

31) This DSO also has designation M42. [6]

a) Identify this DSO. [0.5] Orion Molecular Cloud Complex

b) Identify all image(s) and wavelength(s) of this DSO, [2] Image 11 [0.33], Infrared [0.25], Image 13 [0.33], X-ray [0.25] and Optical [0.25], Image 17 [0.33], Infrared [0.25]

c) Ignoring the constellation, where did this DSO get its name? [1] Star [0.5] lambda Orionis [0.5]

d) In abyssal clays, a maximum in a transition metal isotope can be found millions of years ago. Identify this isotope, and describe the connection between this DSO and the maximum. [2.5, TIEBREAKER 3] Iron-60 [1], the Sun may have passed through the Orion-Eridanus Superbubble [1], enriching older, deep sea sediments [0.5]

32) A certain astronomical body can be modeled using a "lighthouse model" [5]

a) Identify two DSOs on this year's list with an object that fits this description. [1] Crab Nebula [0.5] and The Bone [0.5] also Cas A [0.5]

b) Briefly describe the formation histories of the two DSOs identified in (a). [2] Electron capture supernova [1] and rapidly moving pulsar struck a galactic filament [1] also core collapse supernova [0.5]

c) For each DSO, identify the constellation it is primarily found in. [1] Taurus [0.5] and Sagittarius [0.5] also Cassiopeia [0.5]

d) In which years were these DSOs discovered? [1] 1054 [0.5] and 2025 [0.5] or 1948 [0.5]

33) Bonus! How many images contain DSOs not on this year's list? Which images are these? *Hint: Use process of elimination.* [3] 2 Images [1], Image 3 [1] and Image 16 [1].

These are of the Ring Nebula and Wolf-Rayet 124 (my personal favorite star).

Section C (Astrophysics)

(35 pts total)

1) **White Dwarf Cooling [11].** White Dwarfs and Neutron Stars are the only end states of stars able to be directly observed. Being stellar remnants, they do not have an internal energy source and thus lose their thermal energy through radiative cooling.

To keep this section computationally light, assume

- Uniform density
- Ideal gas Equation of State: $P = \rho \frac{k_B}{\mu m_{proton}} T$ (very generous assumption since a WD is a degenerate gas)

a) The average kinetic energy of a system is given by $\frac{3}{2} N k_B T$, where T will be assumed to be $T_{surface}$ for simplicity and N is the number of particles. Rewrite N in terms of total mass of the star M $[kg]$, average atomic mass μ, and the mass of a proton m_{proton} $[kg]$ and $T_{surface}$. [1]

$$KE\ per\ particle = \frac{3}{2} kT \qquad KE_{System} = \frac{3}{2} NkT,\ N\ particles$$

$$N = \frac{Mass\ of\ System}{Mass\ per\ particle} = \frac{M}{\mu m_H}$$

The rate of temperature loss is (assuming changes in T are small over time):

$$\dot{T} = -\frac{8\pi\sigma m_{proton}}{3k_B a}\left(\frac{\mu}{M_{Star}} R^2 T_{surface}^{\ 4}\right)$$

with the scale factor $a = \frac{T_{core}}{T_{surface}}$

Derivation for $\dot{T}$ (if you're interested):

$$L = -\frac{dE}{dt} = 4\pi\sigma R^2 T^4 = -\frac{d}{dt}\left[\frac{3}{2} a\left(\frac{M_{star}}{\mu m_{proton}}\right) k_B T\right] = -\frac{3}{2} a\left(\frac{M_{star}}{\mu m_{proton}}\right) k_B \frac{dT}{dt}$$

$$\dot{T} = \frac{dT}{dt} = -\frac{8\pi\sigma m_{proton}}{3k_B a}\left(\frac{\mu}{M_{Star}} R^2 T^4\right)$$

The questions below will focus on a white dwarf with the following properties

Mass	$1\, M_\odot$
Radius	$1\, R_\oplus$ (Earth)
$T_{Surface}$	50,000 K
Composition	50% Carbon, 50% Helium

$$m_{proton} = 1.67 \cdot 10^{-27}\, kg$$

b) If Carbon and Helium ions are approximately 12x and 4x the mass of a Hydrogen ion, respectively, what is μ, the average atomic mass relative to a Hydrogen ion? [1]

$$\mu = 0.5(12) + 0.5(4) = 8$$

c) The central pressure of our theoretical WD is $1.9 \cdot 10^{22}\, Pa$. From the ideal gas equation of state, derive a, the scale factor. [2]

$$P = \rho \frac{k_B}{\mu m_{proton}} T \Rightarrow P(r) = \rho \frac{k_B}{\mu m_{proton}} T(r)$$

$$\rho = \frac{1\, M_\odot}{\frac{4}{3}\pi\left(1\, R_\oplus\right)^3}$$

$$P(0) = \rho \frac{k_B}{\mu m_{proton}} T_{core}, \qquad T_{core} = 1.0 \cdot 10^{10}\, K$$

$$a = \frac{1.0 \cdot 10^{10}\, K}{50000\, K} = 2 \cdot 10^{5}$$

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d) What is the rate that our theoretical White Dwarf cools in K/yr? (Sign must be correct) [1]

$$\frac{dT}{dt} = -\frac{8\pi\sigma m_{proton}}{3k_B\left(2 \cdot 10^5\right)}\left(\frac{8}{1.99 \cdot 10^{30}\, [kg]}\left(6378 \cdot 10^3\, [m]\right)^2 \left(50,000\, [K]\right)^4\right)$$

$$= -2.94 \cdot 10^{-13}\, K/s \Rightarrow -9.3 \cdot 10^{-6}\, K/yr$$

Integrating the inverse of the temperature time derivative of our 50% Carbon, 50% Helium white dwarf yields the cooling time:

$$\int_{T_i}^{T_f} \left(\dot{T}\right)^{-1} dT \Rightarrow$$

$$T\,[years] = 2.25 \cdot 10^{23}\left(\frac{1}{T_{final}^{3}} - \frac{1}{T_{initial}^{3}}\right)$$

e) According to the model above, how many years would it take from a newly formed white dwarf to cool from 100,000 K to 150 K (surface temperature of Jupiter)? [1]

$$2.25 \cdot 10^{23}\left[years\ K^3\right]\left(\frac{1}{(150\ K)^3} - \frac{1}{(100{,}000\ K)^3}\right) \Rightarrow$$

$$6.67 \cdot 10^{16}\ years$$

f) The first stars formed and died ~13.8 billion years ago. With an initial temperature of 100,000 K and radius equal to Earth's, what is the maximum possible temperature of the oldest white dwarves now? [1]

$$2.25 \cdot 10^{23}\left[years\ K^3\right]\left(\frac{1}{T_f^{\ 3}} - \frac{1}{(100{,}000\ K)^3}\right) = 13.8 \cdot 10^9\ years$$

$$T_f = 25220\ K$$

Note, the real value is approximately ~3000 - 4000 K

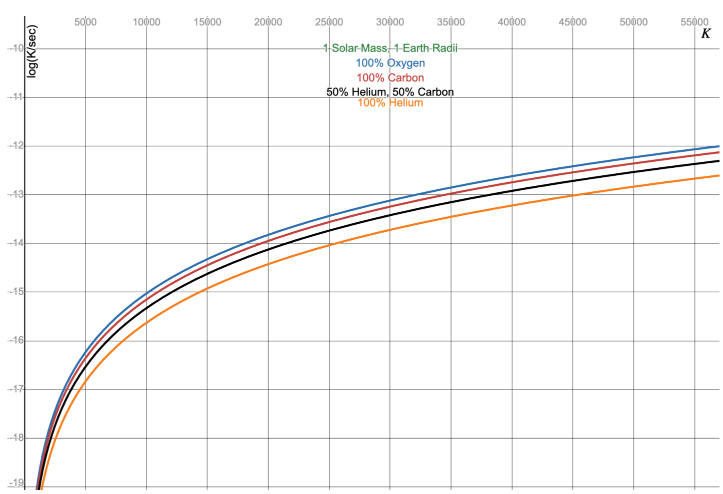
g) What is the maximum possible absolute magnitude of the oldest white dwarves now? Assume Earth Radius. [2]

$$L = 4\pi\sigma\left(6738 \cdot 10^3\ m\right)^2 (25220\ K)^4 = 1.17 \cdot 10^{25}\ W$$

$$M = 4.83 - 2.5 log\left(\frac{1.17 \cdot 10^{25}\ W}{3.9 \cdot 10^{26}\ W}\right) = 8.6$$

h) The previous questions have dealt with an equal proportion Carbon-Oxygen White Dwarf. Would a higher proportion of oxygen increase or decrease the rate that the White Dwarf cools? Why? [2]

1. $\frac{dT}{dt} \propto -\mu$, so (according to our model) a higher proportion of carbon would increase the rate of cooling since it is heavier than carbon, which increases the average atomic weight.
2. A higher particle mass means lower higher specific heat capacity according to the monatomic relation $c_v = \frac{3}{2}\frac{k_B}{\mu m_{proton}}$ and $c_p = \frac{5}{2}\frac{k_B}{\mu m_{proton}}$. (This answer was not needed for full credit, but was included as another reason)



2) **Jupiter Luminosity [14].** Jupiter is the largest giant in the Solar System, accounting for ~70% of planetary mass in the solar system. An odd feature of Jupiter is that it appears to radiate more energy than it receives from the Sun. The following questions will deal with this scenario.

ALL units will be in SI

-	Mass (kg)	Radius (m)	Orbital Radius (AU)
Jupiter	$1.898 \cdot 10^{27}$	$7.0 \cdot 10^{7}$	5.2

a) How is Jupiter able to radiate away more energy than it receives? [1]

Kelvin-Helmholtz Mechanism, gravitational contraction produces heat/energy which is then radiated away

To analytically solve this problem, start with the equation for gravitational potential energy inside a mass m: $U = -\frac{Gm(r)dm}{r^2}$. Assume that the Jupiter is a perfect sphere with uniform density ρ.

b) What is the function $m(r)$ (mass enclosed under a radius r)? [1]

$m(r) = \frac{4}{3}\pi r^3 \rho$

c) What is dm, a thin shell of mass interior to Jupiter, equivalent to? [1]

$dm = 4\pi r^2 \rho dr$ or $4\pi r^2 \rho \Delta r$

Since $t_{freefall} \ll t_{thermal}$, we can estimate Jupiter as being in hydrostatic equilibrium

From the hydrostatic equilibrium equation, pressure as a function of radius is:

$$P(r) = \frac{2}{3} G\pi\rho^2 \left(R^2 - r^2\right)$$

d) What is the ratio of the core pressure of Jupiter compared to the Sun? Use SI Units. [2]

$$P_{Jupiter}(0) = \frac{2}{3} G\pi \left(\frac{M_{Jupiter}}{\frac{4}{3}\pi R_{Jupiter}^{3}}\right)^2 R_{Jupiter}^{2} = 1.19 \cdot 10^{12}\ Pa$$

$$P_{\odot}(0) = \frac{2}{3} G\pi \left(\frac{M_{\odot}}{\frac{4}{3}\pi R_{\odot}^{3}}\right)^2 R_{\odot}^{2} = 1.34 \cdot 10^{14}\ Pa$$

$$\frac{P_{Jupiter}(0)}{P_{\odot}(0)} = 0.00888 \text{ or } 0.8\%$$

e) Assuming the ideal gas equation of state, estimate the central temperature of Jupiter in K. The average mass of a particle in Jupiter is $1.13m_{proton}$. Compare to the ignition of the p-p chain at $10^7\ K$. [2]

$$m_{proton} = 1.674 \cdot 10^{-27}\ kg.$$

$$\text{Gas Constant: } R_{gas} = \frac{k_B}{m_{average}}$$

$$P = \rho RT \Rightarrow P = \rho \frac{k_B}{m_{particle}} T, \quad m_{proton} = 1.67 \cdot 10^{-27}\ kg$$

$$1.19 \cdot 10^{12}\ Pa = \left(\frac{M_{Jupiter}}{\frac{4}{3}\pi R_{Jupiter}^3}\right)\left(\frac{k_B}{1.13 m_{proton}}\right) T_c$$

$$T_c = 1.23 \cdot 10^5\ K$$

$$\frac{T_c}{10^7\ K} = 0.0123 \text{ or } 1.23\%$$

Let's analyze the possibility of hydrogen fusion inside Jupiter.

From Coulomb's Law, the electric force is $k\frac{q_1 q_2}{d^2}$, where $k = 8.99 \cdot 10^9\ Nm^2/C^2$

From Kinetic Theory, the KE per particle is: $\bar{KE} = \frac{3}{2}k_B T$, $\quad k_B$ = Boltzmann Constant

H ion charge: $q = 1.6 \cdot 10^{-19}\ C$

2x H ion radius: $d_0 = 1.75 \cdot 10^{-15}\ m$

f) Bringing two particles from an infinite distance away to to distance d, estimate E_{Fusion}, the energy required to fuse two hydrogen nuclei. [1]

$$E = \vec{F} \cdot \vec{d} \Rightarrow k\frac{q_1 q_2}{d^2} d = k\frac{q_1 q_2}{d}$$

$$E_{Fusion} = E_f - E_i = k\frac{q_1 q_2}{d_0} - k\frac{q_1 q_2}{\infty}, \quad k\frac{q_1 q_2}{\infty} \approx 0$$

$$E_{Fusion} = k\frac{q_1 q_2}{d_0} = \left(8.99 \cdot 10^9\ Nm^2/C^2\right)\frac{(1.6e{-19}\ C)^2}{1.75e{-15}\ m} = 1.32 \cdot 10^{-13}\ J$$

g) What is the ratio of the kinetic energy of a proton in Jupiter's core to E_{Fusion}? [1]

$$\bar{KE} = \frac{3}{2}k_B T_c = \frac{3}{2}k_B\left(1.23 \cdot 10^5\ K\right) = 2.55 \cdot 10^{-18}\ J$$

$$\frac{\bar{KE}}{E_{Fusion}} = 1.93 \cdot 10^{-5} \text{ or } 0.0019\%$$

(note that this does not account for quantum tunneling)

Through some calculus and the Virial Theorem, we get the equation for the gravitational potential energy of a sphere.

$$|U| = \frac{3GM^2}{10R}$$

h) To get a sense of scale, how long would it take for Jupiter to radiate away its entire gravitational potential energy at the Sun's luminosity in years? [2]

$$t = \frac{Energy}{Luminosity} = \frac{3GM_J^{\ 2}}{10R_J}/L_\odot$$

$$L_\odot = 3.9 \cdot 10^{26}\, W,\ M_J = 1.899 \cdot 10^{27}\, kg,\ R_J = 6.9 \cdot 10^{7}\, m$$

$$t = 2.68 \cdot 10^{9}\, s \Rightarrow 84.9\, years$$

i) Jupiter's total "luminosity" (energy reflected from the Sun + its own emission) is roughly $2.078 \cdot 10^{18}\, W$. If Jupiter's geometric albedo is 0.5, respectively, what is the theoretical rate at which Jupiter is shrinking (r_{rate}) in mm/yr? For comparison, the actual rate is estimated to be 2 cm/yr. [3]

The following formulae may be useful.

$$Total\ Energy\ Radiated = Energy\ Reflected + Energy\ Contraction$$

$$Energy\ Contraction = \frac{3GM^2}{10R^2} r_{rate}$$

$$Luminosity\ from\ Sun\ Reflected:\ 0.5\left(\frac{L_{Sun}}{4\pi\left(5.2\, AU \cdot \frac{1.5 \cdot 10^{11}\, m}{1\, AU}\right)^2}\right)\left(\pi R_J^{\ 2}\right)$$

$$= 3.916 \cdot 10^{17}\, W$$

$$Energy\ Radiated\ by\ Jupiter = 2.078 \cdot 10^{18}\, W - 3.916 \cdot 10^{17}\, W = 1.686 \cdot 10^{18}\, W$$

$$1.686 \cdot 10^{18}\, W = \frac{3GM_J^{\ 2}}{10R_J^{\ 2}} r_{rate},\quad r_{rate} = 1.143 \cdot 10^{-10}\, m/s \Rightarrow 3.6\, mm/yr$$

The actual rate is estimated to be 2 cm/year

3) Binary Planet Analysis [10]. For something closer to home, the Pluto-Charon system is unique because of the relatively large mass of the moon, Charon, compared to the larger body. Earth observations reveal that the light curve of the Pluto-Charon System repeats every 6.39 days.

a) (2 pts) Based on astrometric measurements, Charon is observed to be about 0.7” (arcseconds) from Pluto. If the distance from Earth to the Pluto-Charon system is 38.5 AU, what is the distance from Pluto to Charon in kilometers? [1]

$$(1\ AU = 1.5 \cdot 10^{8}\ \frac{km}{AU})$$

$$0.7'' = 206265\left(\frac{x\ [km]}{38.5\ AU \cdot 1.5 \cdot 10^{8}\ \frac{km}{AU}}\right),\ x = 19598.57\ km$$

+1 point for using small angle formula

+1 point for final answer

b) (3 pts) What is the total mass of the Pluto-Charon system in kg? [1]

$$P^2 = \frac{4\pi^2 r^3}{Gm_{total}}$$

$$\left(6.39\ days\ \frac{86400\ s}{1\ day}\right)^2 = \frac{4\pi^2\left(19598.57\ km\ \frac{1000\ m}{1\ km}\right)^3}{Gm_{total}}$$

$$m_{total} = 1.46 \cdot 10^{22}\ kg$$

+1 point for using Kepler’s 3rd Law (any version)

+1 point for correct unit conversions

+1 point for final answer

c) (3 pts) What is the relative velocity between Pluto and Charon in km/s? Assume a circular orbit. [2]

$$v = \sqrt{\frac{Gm_{total}}{r}} = \sqrt{\frac{G\left(1.46 \cdot 10^{22}\ kg\right)}{19598.57\ km\ \frac{1000\ m}{1\ km}}} = 222.91 \Rightarrow 0.22\ km/s$$

+1 point for using circular orbital velocity formula

+1 point for correct unit conversions

+1 point for final answer

d) (3 pts) High angular-resolution imaging has determined that Charon is 0.62” from the barycenter. What is the mass of Charon in Earth Masses? [3]

$$M_{Charon} r_{Charon} = M_{Pluto} r_{Pluto}$$

$$M_{Charon}(0.62") = M_{Pluto}(0.70" - 0.62")$$

$$M_{Pluto} + M_{Charon} = 1.46 \cdot 10^{22}\, kg$$

$$M_{Charon}(0.62") = \left(1.46 \cdot 10^{22}\, kg - M_{Charon}\right)(0.70" - 0.62")$$

$$M_{Charon} = 1.67 \cdot 10^{21}\, kg$$

$$\Rightarrow 2.8 \cdot 10^{-4}\, M_{\oplus}$$

+1 point for usage of conservation of momentum/center of mass equation

+1 for using total mass of system

+1 point for final answer

e) (2 pts) What is the mass of Pluto in Earth Masses? [2]

$$M_{Pluto} = M_{total} - M_{Charon} = 1.46 \cdot 10^{22}\, kg - 1.67 \cdot 10^{21}\, kg = 1.29 \cdot 10^{22}\, kg$$

$$\Rightarrow 2.16 \cdot 10^{-3}\, M_{\oplus}$$

+1 for using total mass of system

+1 point for final answer

f) (1 pts) What is M_{Charon}/M_{Pluto}? For reference, $M_{Moon}/M_{Earth} = 0.0123$. [1]

$$\frac{2.8 \cdot 10^{-4}\, M_{\oplus}}{2.16 \cdot 10^{-3}\, M_{\oplus}} = 0.130$$

+1 point for final answer